

# Extending Propensity Scores: Polychotomous Outcomes, Sample Selection, and Incorrect Model Specification

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## Outline

- Observational studies - the good and the bad
- Propensity scores - an overview
- Extending propensity score methods
  - Polychotomous outcomes
  - Weighted regression instead of sub-sampling
- The errors of ignoring selection bias
  - Propensity score simulations
- Future work

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## Observational Studies

- Observational Study: non-random assignment to treatment
- Why use observational studies?
  - Randomized trials impossible (ethics, design)
  - Randomized trials produce biased results
  - Prevalence of administrative data
- ...and why not *always* use them?
  - Selection Bias
  - RCTs are the “gold standard”
  - RCTs deal with unobserved covariates

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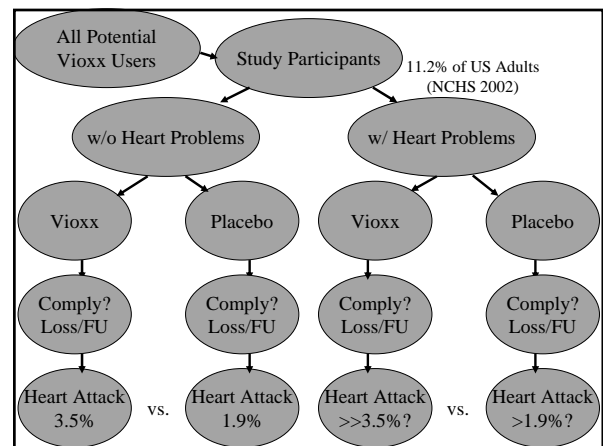
## An Example...

- “Parachute use to prevent death and major trauma related to gravitational challenge: a systematic review of RCTs” *BMJ*, December 2003.
- Objective: To determine if parachutes are effective in preventing trauma and death
- Results: No RCTs found on parachute use. Studies of free fall do not show 100% mortality.
- Conclusion: “The effectiveness of parachutes has not been subjected to rigorous evaluation using RCTs. [Some] criticize the adoption of interventions evaluated using only observational data. Everyone might benefit if the radical protagonists...organized and participated in a double blind, randomized, placebo-controlled, crossover trial of the parachute.”<sup>4</sup>

## RCT Bias

- Psychiatry
  - Excludes patients with multiple symptoms
- Vioxx - New York Times article (Oct 2004)
  - “Good Riddance to a Bad Drug” by Eric J. Topol, Chair, Cardiovascular Medicine, Cleveland Clinic
  - Merck’s new study heart attack/stroke rate:
    - 3.5% on Vioxx
    - 1.9% on Placebo
  - Underestimate since Merck’s trial excluded anyone with known heart disease

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## The Problem with Observational Studies: Selection Bias

- What is selection bias?
  - Those who are in the “treatment” group are not the same as those who get the “control” group
- Uneven baseline covariate distributions
- Need to address this before making valid inferences

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## Simple Illustration

- Mammography for women over 70 years old
  - outcome = early detection of breast cancer
  - treatment = mammography
  - covariate = income
- Assume (hypothetically) that rich women typically get mammography, and poor women typically do not
  - Effect of mammography/income confounded
- Does covariate regression address this?

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## Propensity Score Motivation

- Goal - Identify groups that have similar baseline characteristics, then compare outcome of interest by groups
- Cochran (1968) - Remove bias by stratifying on confounders (90% reduction with 5 strata)
- Rosenbaum and Rubin (1983) - Combine multiple confounders into one measure (propensity score) and use it to stratify

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## Propensity Score Method

- Calculate the conditional probability, or “propensity,” of being in the treatment group (given your covariates)
  - propensity =  $P(\text{treated} \mid \text{covariates})$
- Create groups that are similar to each other in their propensity score by sub-sampling
  - Split into percentiles
  - Matching (nearest, logit, or Mahalanobis)
  - Matching with calipers
  - Greedy algorithm (Parsons)
- Analyze using typical statistical techniques

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## (Some) Applications of PS

- Myocardial Infarction/Coronary Artery Disease
  - Numerous articles (Am J Med, NEJM, Ann Thoracic Surg, JAMA, etc.)
- Psychiatry
  - Lavori, et al - Stats Med 1988, Neuro 1992, J Psy Res 1998
- Mammography
  - Posner, et al - HSORM 2002
- Domestic Abuse
  - Berk, Newton - Am Soc Rev 1985
- Income/Tax Statistics
  - Czajka, et al - J Bus & Econ Stat, 1992
- Gender Bias
  - Zanutto - ASA Proceedings, 2002

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## Recent Research

- Comparing observational studies and RCTs
  - Similar results, when biases are correctly addressed
  - Literature Reviews/Meta-analyses
    - Benson, Hartz. NEJM, 2000
    - Concato, et al. NEJM, 2000
    - Ioannidis, et al. JAMA, 2001

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## Current Research

- Understanding Selection Bias
  - Hernán, Hernández-Díaz, Robins (Epidemiology, 2004)
- Polychotomous Outcomes (Dose-Response)
  - Imbens (Biometrika, 2000)
    - Mental Health Services - Foster (Medical Care, 2003)
  - Imai and Van Dyk (JASA, 2004)
- Matching to Multiple Controls
  - Stuart and Rubin (in progress)
- Missing Data
  - D'Agostino, Rubin, Others via EM, ECM algorithms
- Drop-out prior to completion of trial

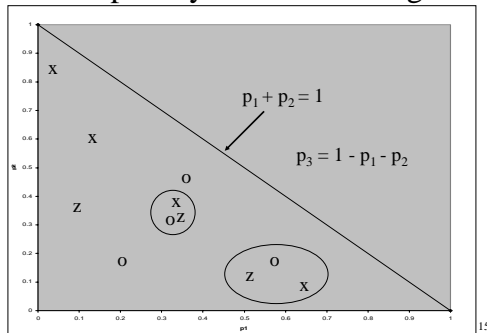
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## Polychotomous Treatments

- Multiple treatment groups
  - Drug A vs. B vs. C
  - Dose-response
- Propensity score analysis of polychotomous treatments
  - Pairwise Logistic Regression
    - k-1 logistic regression equations with one group as the constant reference group
  - Multinomial Regression

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## Polychotomous Multinomial Propensity Score Matching



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## Polychotomous Propensity Scores Example: Respite Analysis

- Ongoing Research
  - w/ Boston Health Care for the Homeless
- Respite unit takes homeless patients not ready to return to streets after hospitalization
  - Treatment groups: respite, home, AMA
- Outcome = 90-day hospital readmission
- Assignment to respite likely associated with patient characteristics, thus we need to consider dealing with selection bias

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## Selection Bias Susceptibility

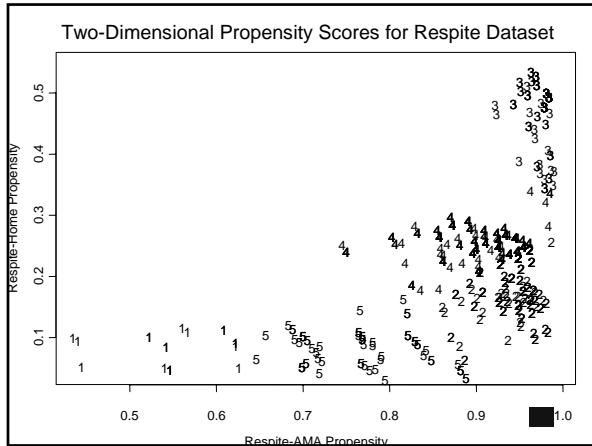
- All variables associated with exposure (respite vs. AMA vs. home) [ $X^2 p < .05$ ]
  - Age (young, middle, old)
  - Race (White, Black, Other)
  - Alcohol Use (yes/no)
  - Drug Use (yes/no)
  - Comorbidity (DCG RR  $<.5$ ,  $.5-1.5$ ,  $>1.5$ )
  - Length of Stay (short, medium, long)
- Many ways of showing or testing covariate balance (Love, JSM 2002)

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## Methods: Two Logistic Regressions

- Calculate the logistic regression equation for the propensity of being in respite:
  - 1.  $P(\text{Respite} \mid \text{Respite or AMA})$
  - 2.  $P(\text{Respite} \mid \text{Respite or Home})$
- Apply these equations to produce two propensity scores for each observation
- Use this two-dimensional vector to quasi-randomize using cluster analysis

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### Results: Two Logistic Regressions

|  |  |  |  |  |  |
|--|--|--|--|--|--|
|  |  |  |  |  |  |
|  |  |  |  |  |  |
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|  |  |  |  |  |  |

- ### Weighted Regression
- Weighted by the ratio of the largest sample size in that cluster, divided by the sample size in that treatment group
  - Weights are then standardized to equal total sample size (default in SAS)
  - Each treatment group is given equal weight in the final analysis

### Example of Weight Calculation

|                | cluster 1 |       |         |                        |
|----------------|-----------|-------|---------|------------------------|
|                | Home      | AMA   | Respite |                        |
| sample size    | 15        | 3     | 1       | ← Total=19 (samp size) |
| total weight   | 15        | 15    | 15      | ← Maximum Samp Size    |
| weight/obs     | 1         | 5     | 15      | ← Weight/sample size   |
| reduced wt     | 0.422     | 2.111 | 6.333   | ← 19/45 * weight/obs   |
| weighted total | 6.333     | 6.333 | 6.333   | ← Total=19, equal wts  |

Similar calculations can be done in other clusters

### Results: Comparing Methods

|  |  |      |  |
|--|--|------|--|
|  |  | OR   |  |
|  |  | 0.61 |  |
|  |  | 1.09 |  |
|  |  | 0.33 |  |
|  |  | 0.60 |  |
|  |  | 0.56 |  |
|  |  | 1.47 |  |

\*Odds Ratio uses "home" as reference group

### Sub-Sampling Concerns

- Sub-sampling results are unstable and very sensitive to randomization (small n/group)

|  |  |         |         |
|--|--|---------|---------|
|  |  | OR      |         |
|  |  | 0.33    |         |
|  |  | 0.60    |         |
|  |  | 0.56    |         |
|  |  | 1.49    |         |
|  |  | invalid | results |
|  |  | invalid | results |
|  |  | 1.41    |         |
|  |  | 2.93    |         |
|  |  | invalid | results |
|  |  | invalid | results |
|  |  | 0.25    |         |
|  |  | 1.60    |         |

## Which Method to Use?

- Sample size potentially too small with sub-sampling
- Randomization can produce inconsistent results
- Weighted method produces consistent results with more precision

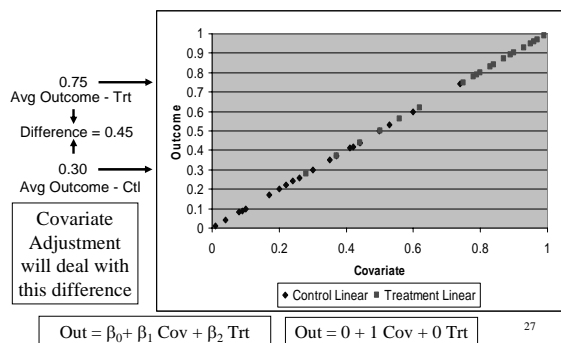
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## Propensity Score Simulations Goals

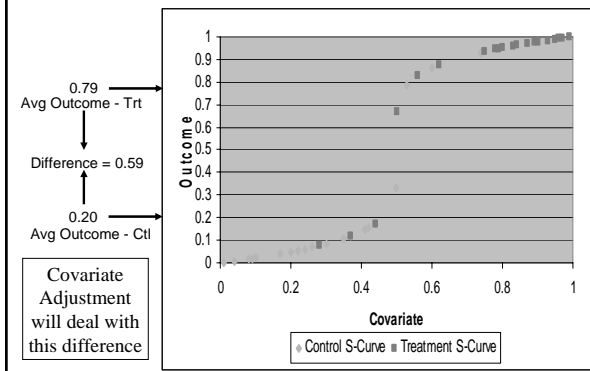
- Understand conditions for which propensity scores are needed
- Assess what goes wrong when models are incorrectly specified
- Demonstrate that stratification helps
- Understand what regression does and doesn't do

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## Correctly Modeled - Linear



## Correctly Modeled - S-Shaped

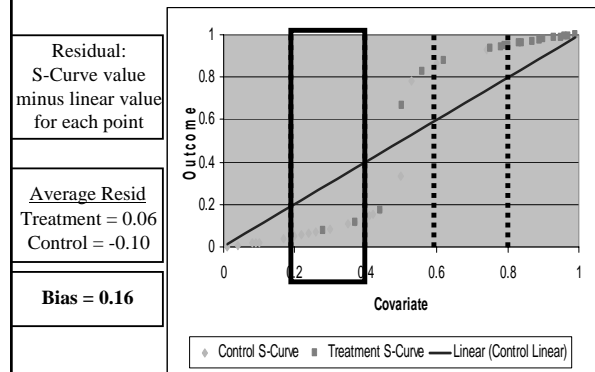


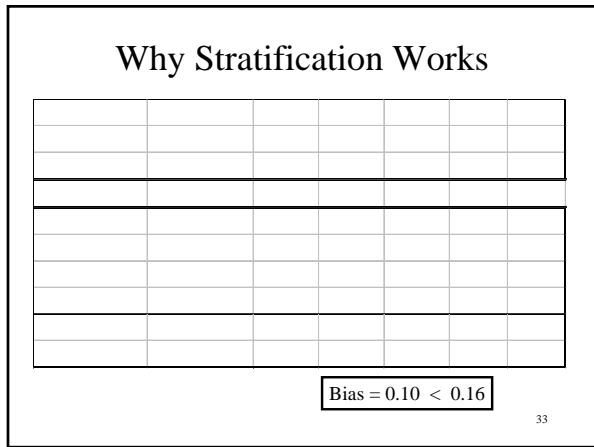
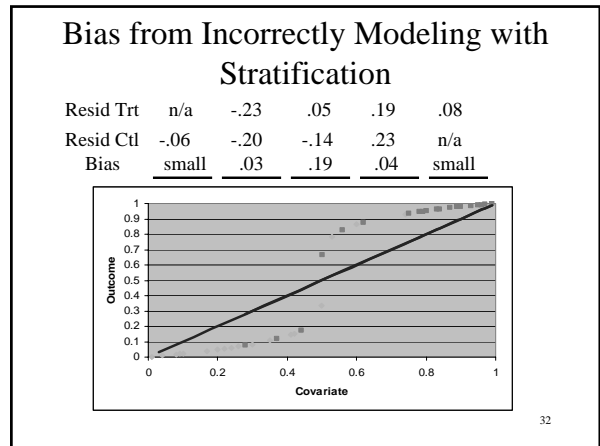
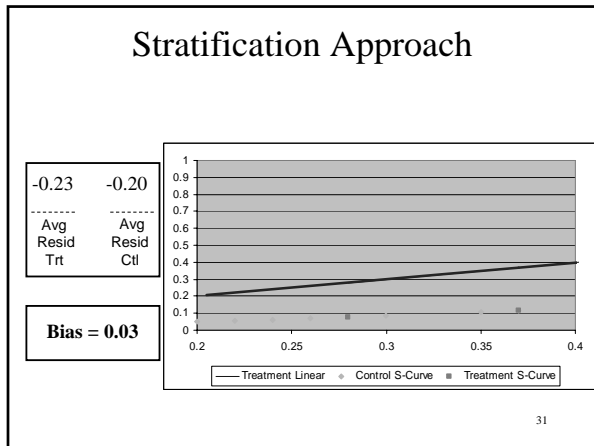
## Incorrect Model Specification

- What does it mean to be incorrectly specified?
  - Regression Model:  $ED \sim Mam + Income + \epsilon$
  - Examples of incorrect model specification:
    - $ED \sim Mam + (Income)^2 + \epsilon$
    - $ED \sim Mam + (Income)^4 + \epsilon$
    - $ED \sim Mam + Income + Education + \epsilon$
  - Generally:  $ED \sim Mam + f(X)\beta + g(Z)\gamma + \epsilon$   
X are associated with Mam, Z are not

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## Incorrect Model Specification

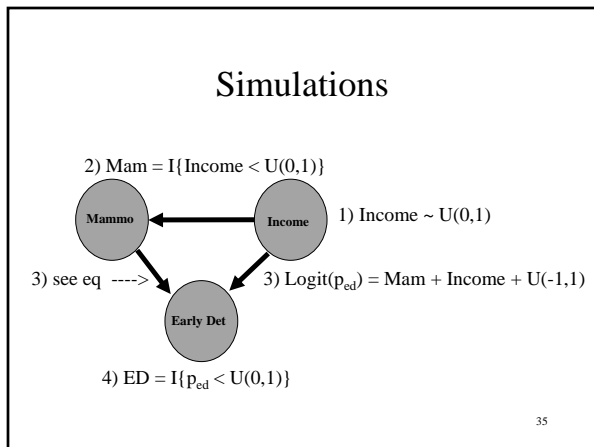




### Regression with Covariates

- What it does:
  - Adjusts for correctly specified model
- What it doesn't do:
  - Adjusts for incorrectly specified model
  - Deal with unobserved covariates

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### Simulations: Quasi-Randomization

- Examining Mean Income

|  | n         | mam  | no mam | t    |
|--|-----------|------|--------|------|
|  | 1,000,000 | 0.67 | 0.33   | -708 |
|  | 520,024   | 0.51 | 0.49   | -31  |

- Susceptible to selection bias

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## Simulations: Comparing Bias

Underlying (true) model is ED ~ Mam + Inc

| Regression Model |      | OR <sub>mam</sub> |
|------------------|------|-------------------|
| ED ~ Mam + Inc   | regr | 2.65              |
|                  | w/PS | 2.66              |
| ED ~ Mam         | regr | 3.61              |
|                  | w/PS | 2.69              |

Adjustment is necessary when incorrect modeling might be present

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## Simulations: Incorrectly Specified Model

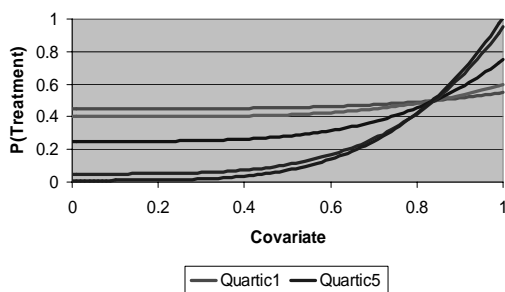
|  | No PS | PS   |
|--|-------|------|
|  | 2.65  | 2.66 |
|  | 2.81  | 2.70 |
|  | 3.17  | 2.78 |

Regression model always ED ~ Mammography + Income

The more incorrect the model, the more the bias

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## Varying Strength of Association



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## Simulations: Strength of Propensity Score Model

|           | No PS | c (PS) | w/ PS |
|-----------|-------|--------|-------|
| Quartic 1 | 2.73  | 0.55   | 2.70  |
| Quartic 2 | 2.79  | 0.65   | 2.72  |
| Quartic 3 | 2.95  | 0.78   | 2.76  |
| Quartic 4 | 3.03  | 0.84   | 2.78  |
| Quartic 5 | 3.17  | 0.92   | 2.78  |

Conclusion 1: The stronger the predictive ability of the propensity model, the more susceptible to selection bias

Conclusion 2: Using the c-statistic from the propensity score model may be better than looking at the bivariate distributions to assess the need for propensity score methods

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## Future Research

- Using other measures besides c-statistic
- Varying the strength of the relationship between exposure(s) and outcome
- Including covariates into the simulation models (like education) and varying parameters to determine their effects on biases and outcomes

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## Summary/Conclusions

- Methods for dealing with bias in observational studies are important
- We have seen two extensions to propensity score methods
  - polychotmous treatments
  - weighted techniques
- When considering propensity score methods, we should...
  - Examine the predictive ability of assignment to treatment
  - (Perhaps) examine the relationship between exposure and outcome

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## Thank You!

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